SAMPLE

The Dirac Equation

Relativistic Quantum Mechanics and Antimatter

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Introduction

In 1928 a twenty-five-year-old Cambridge physicist set out to remove an inconsistency that no one else regarded as urgent, and in a few weeks of solitary work produced the equation that carries his name. Paul Dirac's problem was that the quantum mechanics of the day ignored Einstein: the Schrödinger equation, the new mechanics' working engine, treated time as special and space as ordinary, and a description of the electron written that way could not survive the test of high speeds. Dirac wanted one mechanics that was quantum and relativistic at once, and the equation he found was so strict a marriage of the two theories that it demanded things no one had asked for. It demanded that the electron spin. It fixed the electron's magnetism to the value experiments measured. And it carried, written into its negative-energy solutions, the prediction that every particle must have an oppositely charged mirror twin, a prediction confirmed in the cosmic rays four years later and vindicated in Stockholm the year after that.

The story of the Dirac equation is therefore the story of modern physics in miniature: the founding of quantum mechanics in the 1920s, the reconciliation with relativity, the discovery of antimatter, the construction of quantum electrodynamics, the most exactingly checked theory in the history of science, and, in the twenty-first century, the surprise that the equation had been hiding all along in pencils and crystals. This book tells it whole, in plain prose and with the mathematics carried in words, because the symbols are not the story. What the reader needs to carry is the shape of the thing: a single law, found by demanding that nature be consistent, that turned out to describe more than its author imagined and less than its author feared.

The chapters run in the order of the logic and the history together. First the problem the equation solved, then its construction, its early triumphs, its crisis of negative energy, and the antimatter answer. Then the working life of the equation: the field it defines, the symmetries it obeys, its behavior in curved spacetime where black holes live, its condensed-matter second career, its quiet role in the color of gold and the scanning of hospital patients. The book closes with Dirac himself, the strangest man in the story of physics, and the plaque in Westminster Abbey where the equation, four symbols long, is set in stone beside the memorial of Newton.

A note on the plan of the chapters, for the reader who likes the map before the road. The book opens with the problem the equation solved and the derivation that solved it, follows the consequences through spin, magnetism, and the negative-energy crisis, and watches the crisis turn into antimatter and the antimatter into the positron of the cloud chamber. The working chapters then

take the equation through its careers: the field, the symmetry structure, the instruments, the curved spacetime where black holes shine, and the standards, crystals, chemistries, and clocks it quietly holds together. The closing chapters carry the story outward, to the early universe's missing antimatter and the laboratories that trap it, and then inward, to the silent man at the center of it all, and the reader who wants the book in a single sentence holds it here: one equation, found by demanding consistency, that ended up describing more of the world than any other single sentence physics has produced.

Chapter 1: The Discovery of the Dirac Equation

The discovery of the Dirac equation begins with the two revolutions that made it necessary, and the reader needs both in view to see what Dirac saw. The first revolution was relativity. Einstein's special relativity of 1905 had fixed the rules for fast-moving objects, among them the most famous constraint in physics, that nothing travels faster than light, and the most consequential for what follows, that time and space enter the laws of physics on an equal footing, mixed together for moving observers like the ingredients of one substance. The second revolution was quantum mechanics, assembled in 1925 and 1926 in two rival forms: matrix mechanics, built by Heisenberg, Born, and Jordan, and wave mechanics, built by Schrödinger, which described a particle by a wave and its behavior by a wave equation. By 1927 the two forms were understood as one mechanics, and the new physics had its engine.

The engine, however, was non-relativistic. Schrödinger's equation treats time as a rate of change but space as a plain stage, and the balance between them is wrong for anything moving near the speed of light. The obvious fix, attempted within months of the original equation, was to make the time and space parts enter symmetrically; the result, known as the Klein-Gordon equation after its inventors, was mathematically correct and physically embarrassing. Its troubles were two. It gave up the probability interpretation that made the new mechanics work: the quantity that should represent the probability of finding a particle can go negative in the Klein- Gordon theory, and probability cannot be negative. And it said nothing about spin, the intrinsic rotation that electrons demonstrably carry and that all the fine structure of atomic spectra depends on. The equation was right the way a broken clock is right about nothing: formally consistent, physically unusable.

Dirac saw the problem as an aesthetic one before it was a technical one, and aesthetics drove everything he did. He was working at Cambridge in the immediate circle of the new mechanics' arrival, a young man who had learned the theory as it was being made and judged it, as he judged all physics, by its beauty and its consistency. The inconsistency that bothered him was the mismatch: the most successful theory of matter ever written refused to speak the language of the most successful theory of motion. His aim, stated simply, was an equation that took the quantum wave and made it relativistic, keeping probability positive and time and space equal, and he found it by a route so short it has become the standard demonstration of what theoretical physics is.

The route was the square root. The Klein-Gordon equation, in its essence, relates the energy of a particle to its momentum in the pattern relativity demands, and the pattern is quadratic: energy squared equals mass squared

plus momentum squared, in the appropriate units. Schrödinger's equation, by contrast, is linear: it advances the wave in time directly, one derivative, no squaring. Dirac asked what happens if one insists on both: a wave equation linear in the time derivative, as quantum mechanics requires, whose square reproduces the relativistic pattern, as relativity requires. The demand is not a guess; it is a straitjacket. Work out what the coefficients of the linear equation must be for its square to come out clean, and the arithmetic forces a very specific algebraic structure on them: the coefficients must all anticommute, each pair multiplying to change sign, a structure of objects no one had put in a wave equation before.

The algebra that answered Dirac's demand was, unknown to him, already in the mathematics of the 1880s, where it had been studied for its own sake as a way to encode geometry in algebra. Dirac reinvented it for physics, found the smallest set of objects that satisfied his straitjacket, and wrote the equation with them. The wavefunction the equation demands is not Schrödinger's one-component wave but a column of four complex numbers, four components where quantum mechanics had used one and Pauli's spin theory had used two. Dirac had no physical interpretation to offer for the doubling; he simply followed the mathematics and let the interpretation come. The equation was published in 1928 in the Proceedings of the Royal Society, under the plain title The Quantum Theory of the Electron, and physics has been organized around it ever since.

The Cambridge that produced the derivation deserves its paragraph, because the setting was not incidental. Dirac had arrived there from Bristol in 1923, and was supervised in his doctoral work by Ralph Fowler, the physicist whose household and circle connected Cambridge to the new quantum theory as it was being invented abroad, and it was through that channel that the papers of Heisenberg, Born, and Schrödinger reached Dirac within months of their writing, sometimes within days. By 1927 the trade's centers were Copenhagen, Gottingen, and Cambridge, and of the three, Cambridge was the quietest and the most mathematical, the environment suited to a man who worked alone, distrusted conversation as a source of physics, and held that the theory's development was best conducted in silence and on paper. The equation that came out of that silence was, in consequence, the most Cambridge thing in the new physics: a result with no experimental prompting, produced by consistency alone, from a school that regarded itself as the keeper of the mathematics.

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ABOUT THE AUTHOR

Aira Astralis is a science writer specializing in astronomy and space exploration. Her work spans the full breadth of humanity's relationship with the cosmos -- from the earliest naked-eye observations to the engineering marvels of modern space missions.

Her books are built on a foundation of primary sources: peer-reviewed research, mission archives, and direct engagement with the scientific community. She writes for readers who want to understand not just what we know about the universe, but how we came to know it.

She believes that the most important frontier is not out there but in the gap between what scientists discover and what the public understands. Her books exist to close that gap.