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Quantum Entanglement

Spooky Action at a Distance

Aira Astralis

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Chapter 1: The Discovery of Quantum Entanglement

Their thought experiment considered two systems that interact and then separate, and they argued that, afterward, quantum mechanics could not describe the two systems individually. Shortly after this paper appeared, Erwin Schrödinger wrote a letter to Einstein in German in which he used the word Verschränkung (translated by himself as entanglement) to describe situations like that of the EPR scenario.

Schrödinger followed up with a full paper defining and discussing the notion of entanglement, saying "I would not call [entanglement] one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." Like Einstein, Schrödinger was dissatisfied with the concept of entanglement, because it seemed to violate the speed limit on the transmission of information implicit in the theory of relativity.

Einstein later disparaged quantum mechanics for seemingly exhibiting "spukhafte Fernwirkung", or "spooky action at a distance", meaning the acquisition of a value of a property at one location resulting from a measurement at a distant location. In 1946, John Archibald Wheeler suggested studying the polarization of pairs of gamma-ray photons produced by electron–positron annihilation.

Chien-Shiung Wu and Irving Shaknov carried out this experiment in 1949, thereby demonstrating that the entangled particle pairs considered by EPR could be created in the laboratory.

Despite Schrödinger's claim of its importance, little work on entanglement was published for decades after his paper was published. In 1964 John S. Bell demonstrated an upper limit, seen in Bell's inequality, regarding the strength of correlations that can be produced in any theory obeying local realism, and showed that quantum theory predicts violations of this limit for certain entangled systems.

His inequality is experimentally testable, and there have been numerous relevant experiments, starting with the pioneering work of Stuart Freedman and John Clauser in 1972, and Alain Aspect's experiments in 1982. While Bell actively discouraged students from pursuing work like his as too esoteric, after a talk at Oxford a student named Artur Ekert suggested that the violation of a Bell inequality could be used as a resource for communication.

Ekert followed up by publishing a quantum key distribution protocol called E91 based on it.

In 1992, the entanglement concept was leveraged to propose quantum teleportation, an effect that was realized experimentally in 1997. Beginning in the mid-1990s, Anton Zeilinger used the generation of entanglement via parametric down-conversion to develop entanglement swapping and demonstrate quantum cryptography with entangled photons.

In 2022, the Nobel committee in Stockholm gave its physics prize to Aspect, Clauser, and Zeilinger, citing "experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science".

Just as energy is a resource that facilitates mechanical operations, entanglement is a resource that facilitates performing tasks that involve communication and computation. The mathematical definition of entanglement can be paraphrased as saying that maximal knowledge about the whole of a system does not imply maximal knowledge about the individual parts of that system.

If the quantum state that describes a pair of particles is entangled, then the results of measurements upon one half of the pair can be strongly correlated with the results of measurements upon the other. However, entanglement is not the same as "correlation" as understood in classical probability theory and in daily life.

Instead, entanglement can be thought of as potential correlation that can be used to generate actual correlation in an appropriate experiment.

The correlations generated from an entangled quantum state cannot in general be replicated by classical probability. An example of entanglement is a subatomic particle that decays into an entangled pair of other particles.

The decay events obey the various conservation laws, and as a result, the measurement outcomes of one daughter particle must be highly correlated with the measurement outcomes of the other daughter particle (so that the total momenta, angular momenta, energy, and so forth remains roughly the same before and after this process).

For instance, a spin-zero particle could decay into a pair of spin-1/2 particles. If there is no orbital angular momentum, the total spin angular momentum after this decay must be zero (by the conservation of angular momentum). Whenever the first particle is measured to be spin up on some axis, the other, when measured on the same axis, is always found to be spin down.

This is called the spin anti-correlated case and the pair is said to be in the singlet state.

Perfect anti-correlations like this could be explained by "hidden variables" within the particles. For example, we could hypothesize that the particles are made in pairs such that one carries a value of "up" while the other carries a value of "down". Then, knowing the result of the spin measurement upon one particle, we could predict that the other will have the opposite value.

Bell illustrated this with a story about a colleague, Bertlmann, who always wore socks with mismatching colors.

Quantum systems can become entangled through various types of interactions.

For some ways in which entanglement may be achieved for experimental purposes, see the section below on methods. Entanglement is broken when the entangled particles decohere through interaction with the environment; for example, when a

measurement is made. In more detail, this process involves the particles becoming entangled with the environment, as a consequence of which, the quantum state describing the particles themselves is no longer entangled.

Mathematically, an entangled system can be defined to be one whose quantum state cannot be factored as a product of states of its local constituents; that is to say, they are not individual particles but are an inseparable whole. When entanglement is present, one constituent cannot be fully described without considering the other(s).

The state of a composite system is always expressible as a sum, or superposition, of products of states of local constituents; it is entangled if this sum cannot be written as a single product term. The singlet state described above is the basis for one version of the EPR paradox. In this variant, introduced by David Bohm, a source emits particles and sends them in opposite directions.

The state describing each pair is entangled.

The argument that made entanglement famous began in 1935. Einstein, Boris Podolsky, and Nathan Rosen asked whether quantum mechanics could be complete when it allowed a measurement on one member of a separated pair to fix what could be predicted about the other. Their conclusion favored hidden information, properties carried by the particles but omitted by the wavefunction. Erwin Schrödinger answered in the same year, coined the German term that became entanglement, and identified the phenomenon not as a minor oddity but as the characteristic feature of quantum theory. The disagreement then waited nearly thirty years for a test. John Bell's 1964 inequality transformed the philosophical dispute into an experimental choice: local hidden-variable theories obey one numerical limit, while quantum entanglement can violate it. That transformation, from interpretation to number, is the decisive discovery in the subject's history.

Chapter 2: Understanding Quantum Entanglement

In the standard textbook presentation of quantum mechanics, performing a spin measurement on one of the particles causes the wave function for the whole pair to collapse into a state in which each particle has a definite spin (either up or down) along the axis of measurement. The outcome is random, with each possibility having a probability of 50%.

However, if both spins are measured along the same axis, they are found to be anti-correlated.

This means that the random outcome of the measurement made on one particle seems to have been transmitted to the other, so that it can make the "right choice" when it too is measured. The distance and timing of the measurements can be chosen so as to make the interval between the two measurements spacelike, hence, any causal effect connecting the events would have to travel faster than light.

According to the principles of special relativity, it is not possible for any information to travel between two such measuring events. It is not even possible to say which of the

measurements came first. For two spacelike separated events x1 and x2 there are inertial frames in which x1 is first and others in which x2 is first.

Therefore, the correlation between the two measurements cannot be explained as one measurement determining the other: different observers would disagree about the role of cause and effect.

A possible resolution to the paradox is to assume that quantum theory is incomplete, and the result of measurements depends on predetermined "hidden variables". The state of the particles being measured contains some hidden variables, whose values effectively determine, right from the moment of separation, what the outcomes of the spin measurements are going to be.

This would mean that each particle carries all the required information with it, and nothing needs to be transmitted from one particle to the other at the time of measurement. Einstein and others originally believed this was the only way out of the paradox, and the accepted quantum mechanical description (with a random measurement outcome) must be incomplete.

Local hidden variable theories fail when measurements of the spin of entangled particles along different axes are considered.

If a large number of pairs of such measurements are made (on a large number of pairs of entangled particles), then statistically, if the local realist or hidden variables view were correct, the results would always satisfy Bell's inequality. A number of experiments have shown in practice that Bell's inequality is not satisfied.

The fundamental issue about measuring spin along different axes is that these measurements cannot have definite values at the same time—they are incompatible in the sense that these measurements' maximum simultaneous precision is constrained by the uncertainty principle.

This is contrary to what is found in classical physics, where any number of properties can be measured simultaneously with arbitrary accuracy. It has been proven mathematically that compatible measurements cannot show Bell-inequality-violating correlations, and thus entanglement is a fundamentally non-classical phenomenon. As discussed above, entanglement is necessary to produce a violation of a Bell inequality.

However, the mere presence of entanglement alone is insufficient, as Bell himself noted in his 1964 paper.

This is demonstrated, for example, by Werner states, which are a family of states describing pairs of particles. For appropriate choices of the key parameter that identifies a given Werner state within the full set thereof, the Werner states exhibit entanglement. Yet pairs of particles described by Werner states always admit a local hidden variable model.

In other words, these states cannot power the violation of a Bell inequality, despite possessing entanglement. This can be generalized from pairs of particles to larger collections as well.

The violation of Bell inequalities is often called quantum nonlocality. This term is not without controversy. It is sometimes argued that using the term nonlocality carries the unwarranted implication that the violation of Bell inequalities must be explained by physical, faster-than-light signals. In other words, the failure of local hidden-variable models to reproduce quantum mechanics is not necessarily a sign of true nonlocality in quantum mechanics itself.

Despite these reservations, the term nonlocality has become a widespread convention.

The term nonlocality is sometimes applied to other concepts besides the nonexistence of a local hidden-variable model, such as whether states can be distinguished by local measurements. Moreover, quantum field theory is often said to be local because observables defined within spacetime regions that are spacelike separated must commute. These other uses of local and nonlocal are not discussed further here.

The following subsections use the formalism and theoretical framework developed in the articles bra–ket notation and mathematical formulation of quantum mechanics.

Consider two arbitrary quantum systems A and B, with respective Hilbert spaces HA and HB.

If the first system is in state

However, not all states of the composite system are separable. Fix a basis

If a state is inseparable, it is called an 'entangled state'.

Therefore, a bipartite pure state $\rho \in H_A \otimes H_B$ is said to be a maximally entangled state if the reduced state of each subsystem of ρ is the diagonal matrix

For two bosonic modes, a NOON state is

It was later found that Simon's condition is also necessary and sufficient for

The mathematical expression for the swapping process is:

Quantum metrology is the study of making high-resolution and highly sensitive measurements of physical parameters using quantum theory to describe the physical systems, particularly exploiting quantum entanglement and quantum squeezing.

This field promises to develop measurement techniques that give better precision than the same measurement performed in a classical framework. Together with quantum hypothesis testing, it represents an important theoretical model at the basis of quantum sensing.

E.g., there are examples when

For example, given two basis vectors

If in the entangled state given above Alice makes a measurement in the

By the spectral theorem, such a matrix takes the general form:

We can interpret ρ as representing an ensemble where

For example, it could produce two populations of electrons: one with state

That is, it has the general form

This is self-adjoint and positive and has trace 1. Extending the definition of separability from the pure case, we say that a mixed state is separable if it can be written as

It is inseparable if for any vectors

For example, given two basis vectors

A useful understanding begins with the pair rather than either member. An entangled pair has a well-defined joint state even when neither part has a complete state of its own. Measuring one part does not reveal an instruction that was simply waiting inside it; the statistical pattern across many paired measurements depends on how both laboratories choose to measure. No single result carries a controllable message, so relativity remains intact, but the combined records display correlations stronger than any local instruction set permits. This distinction between an individual outcome and a statistical correlation prevents the two common mistakes: treating entanglement as ordinary synchronization, and treating it as faster-than-light communication. It is neither. It is a new structure of joint probability, one that belongs to the composite system and cannot be reduced to separate descriptions of its parts.

The laboratory recipes deserve their own description, because making entanglement is now routine craft. The workhorse is the nonlinear crystal: pump it with a bright laser at one frequency, and now and then a pump photon splits inside it into two lower-energy photons whose polarizations and momenta are locked together, a process called spontaneous parametric down-conversion, which produces entangled pairs by the million at room temperature and is the source behind most optics demonstrations. In ion traps, entanglement is manufactured instead by driving two chilled ions with a shared pulse of laser light, so that the collective vibration of the pair carries the correlation; in superconducting circuits, microwave pulses on coupled resonators do the same job in a chip cooled to a few thousandths of a degree above absolute zero. Each platform makes the same physics at a different scale, and the fact that photons, ions, and circuits all show the identical violation statistics is itself evidence that entanglement is a property of nature rather than an artifact of any apparatus.

Chapter 3: Key Characteristics of Quantum Entanglement

Moreover, when measurements of the entangled particles are made in moving relativistic reference frames, in which each measurement (in its own relativistic time frame) occurs before the other, the measurement results remain correlated. This term is not without controversy. Another way to say this is that while the von Neumann

entropy of the whole state is zero (as it is for any pure state), the entropy of the subsystems is greater than zero. In this sense, the systems are "entangled".

For example, given two basis vectors

If the composite system is in this state, it is impossible to attribute to either system A or system B a definite pure state. In this sense, the systems are "entangled".

Now suppose Alice is an observer for system A, and Bob is an observer for system B.

If she obtains the outcome 0, then she can predict with certainty that Bob's result will be 1.

In other words, the results of measurements on the two qubits will be perfectly anti-correlated. This remains true even if the systems A and B are spatially separated. This is the foundation of the EPR paradox. The outcome of Alice's measurement is random.

Alice cannot decide which state to collapse the composite system into, and therefore cannot transmit information to Bob by acting on her system.

Causality is thus preserved, in this particular scheme. For the general argument, see no-communication theorem. As mentioned above, a state of a quantum system is given by a unit vector in a Hilbert space.

More generally, if one has less information about the system, then one calls it an 'ensemble' and describes it by a density matrix, which is a positive-semidefinite matrix, or a trace class when the state space is infinite-dimensional, and which has trace 1.

When a mixed state has rank 1, it therefore describes a 'pure ensemble'. When there is less than total information about the state of a quantum system we need density matrices to represent the state. Experimentally, a mixed ensemble might be realized as follows.

Consider a "black box" apparatus that spits electrons towards an observer. The electrons' Hilbert spaces are identical. The apparatus might produce electrons that are all in the same state; in this case, the electrons received by the observer are then a pure ensemble. However, the apparatus could produce electrons in different states.

Generally, this is a mixed ensemble, as there can be any number of populations, each corresponding to a different state.

A basic task of quantum metrology is estimating the parameter

Typically, the system is composed of many particles, and the Hamiltonian is a sum of single-particle terms

In this case, there is no interaction between the particles, and we talk about linear interferometers.

The achievable precision is bounded from below by the quantum Cramér-Rao bound as

One example of note is the use of the NOON state in a Mach–Zehnder interferometer to perform accurate phase measurements.

A similar effect can be produced using less exotic states such as squeezed states. In quantum illumination protocols, two-mode squeezed states are widely studied to overcome the limit of classical states represented in coherent states. In atomic ensembles, spin squeezed states can be used for phase measurements. An important application of quantum metrology is the detection of gravitational waves in projects such as LIGO and Virgo.

Advanced LIGO has implemented frequency-dependent quantum squeezing to reduce quantum noise (including shot noise and radiation pressure noise) and surpass the standard quantum limit. This technology has been deployed since the start of the O4 observing run in May 2023, providing broadband sensitivity improvements and enabling more precise measurements of spacetime distortions. Quantum metrology is extending into biological systems by engineering quantum effects in biomolecules.

A 2026 study from the University of Oxford demonstrated magneto-sensitive fluorescent proteins (MFPs) that use quantum spin resonance to interact with magnetic fields and radio waves via light-triggered quantum processes. This enables quantum-enhanced sensing for molecular imaging and biological detection, linking quantum metrology to biotechnology. A central question of quantum metrology is how the precision, i.e., the variance of the parameter estimation, scales with the number of particles.

Classical interferometers cannot overcome the shot-noise limit.

This limit is also frequently called standard quantum limit (SQL)

Shot-noise limit is known to be asymptotically achievable using coherent states and homodyne detection. Quantum metrology can reach the Heisenberg limit given by

Since a mixed state ρ is a probability distribution over an ensemble, this leads naturally to the definition of the von Neumann entropy:

Since an event of probability 0 should not contribute to the entropy, and given that

Let the state of the composite system be

The state of A is the partial trace of ρT over the basis of system B:

. The sum occurs over

For example, the reduced density matrix of A for the entangled state

In contrast, the density matrix of A for the pure product state

Three characteristics separate entanglement from familiar correlation. First, it is basis-dependent in its appearance but not in its existence: the same pair can show perfect agreement in one measurement basis and a patterned disagreement in another, while remaining one inseparable state. Second, it is monogamous. A system maximally entangled with one partner cannot be equally entangled with a third, a

restriction that later becomes a security guarantee in quantum cryptography. Third, it is fragile under uncontrolled interaction. When either partner becomes entangled with its environment, the useful two-party coherence disperses into a larger system, a process described as decoherence. These properties give the phenomenon its double character. Entanglement is stronger than classical correlation, constrained more tightly than classical sharing, and easier to destroy than an ordinary stored bit. Every practical design is an attempt to exploit the first two facts before the third one wins.

Chapter 4: The Science Behind Quantum Entanglement

The above example is one of four Bell states, which are (maximally) entangled pure states (pure states of the HA ⊗ HB space, but which cannot be separated into pure states of each HA and HB).

Following the definition above, for a bipartite composite system, mixed states are just density matrices on HA ⊗ HB. That is, it has the general form k

In other words, a state is separable if it is a probability distribution over uncorrelated states, or product states. A state is then said to be entangled if it is not separable.

In general, finding out whether or not a mixed state is entangled is considered difficult.

If the first system is in state

For the 2 × 2 and 2 × 3 cases, a necessary and sufficient criterion for separability is given by the famous Positive Partial Transpose (PPT) condition. The idea of a reduced density matrix was introduced by Paul Dirac in 1930. Consider as above systems A and B each with a Hilbert space HA, HB. However, it still is possible to associate a density matrix. Let T. The sum occurs over dim

Colloquially, we "trace out" or "trace over" system B to obtain the reduced density matrix on A. This demonstrates that the reduced density matrix for an entangled pure ensemble is a mixed ensemble.

In general, a bipartite pure state ρ is entangled if and only if its reduced states are mixed rather than pure. In quantum information theory, entangled states are considered a 'resource', i.e., something costly to produce and that allows implementing valuable transformations.

The setting in which this perspective is most evident is that of "distant labs", i.e., two quantum systems labelled "A" and "B" on each of which arbitrary quantum operations can be performed, but which do not interact with each other quantum mechanically.

The only interaction allowed is the exchange of classical information, which combined with the most general local quantum operations gives rise to the class of operations called LOCC (local operations and classical communication).

The science behind entanglement can be stated without writing its equations. Quantum states combine through superposition, and composite systems combine through a tensor product, a rule that creates possibilities unavailable to either system alone. Some joint states can be factored into one state for Alice's system and one for Bob's; those are separable. Others cannot be factored, no matter which local descriptions are chosen; those are entangled. The Schmidt decomposition gives every pure two-party state a standard form and turns the number and weights of its paired terms into a measure of entanglement. Mixed states, which include classical uncertainty as well as quantum coherence, require subtler tests. Density matrices, partial traces, and entanglement witnesses are the laboratory's mathematical tools, each answering a practical question: what does one subsystem look like, is the joint state inseparable, and how much useful entanglement remains?

A Bell state provides the simplest example of the structure. Prepare two qubits so that the only allowed joint alternatives are both zero and both one, retained in coherent superposition. Measuring either qubit alone gives an unpredictable result. Measuring both in the same basis gives matching results every time. Rotate both measurement bases and the matching pattern persists in the way quantum theory predicts; rotate them differently and the correlation changes continuously. No fixed list of local answers can reproduce every setting at once. The science lies in that counterfactual reach across settings, not in any single dramatic pair of outcomes.

For mixed states, the boundary between quantum and classical correlation becomes harder to locate. A laboratory may prepare different pure states with ordinary probabilities, and the resulting density matrix can look correlated without being entangled. The partial-transpose test gives a decisive criterion for the smallest bipartite dimensions, while entanglement witnesses identify broader families by measuring an observable whose value no separable state can attain. Measures such as entropy of entanglement, concurrence, negativity, and entanglement of formation quantify different operational questions. No one measure answers them all, especially for mixed and multipartite systems. This plurality is not a weakness. It reflects the fact that entanglement is defined by what separated parties can and cannot accomplish under specified operations.

The no-signaling rule completes the formal picture. Alice's choice of measurement changes how the joint correlations are sorted, but Bob's local statistics remain the same until Alice's classical record arrives. Quantum theory is nonlocal in Bell's precise sense and causal in relativity's operational sense. Holding both statements at once is essential. The theory denies local hidden explanations while denying any controllable superluminal channel, a balance that every successful interpretation and every quantum protocol must preserve.

Chapter 5: Observation and Measurement of Quantum Entanglement

These operations do not allow the production of entangled states between systems A

ABOUT THE AUTHOR

Aira Astralis is a science writer specializing in astronomy and space exploration. Her work spans the full breadth of humanity's relationship with the cosmos -- from the earliest naked-eye observations to the engineering marvels of modern space missions.

Her books are built on a foundation of primary sources: peer-reviewed research, mission archives, and direct engagement with the scientific community. She writes for readers who want to understand not just what we know about the universe, but how we came to know it.

She believes that the most important frontier is not out there but in the gap between what scientists discover and what the public understands. Her books exist to close that gap.